Digital SAT Deep Dive: Data Analysis, Geometry, and Trigonometry

A free, unofficial study guide to Problem-Solving and Data Analysis (about 15% of Math) and Geometry and Trigonometry (about 15%). Practice with official Bluebook tests and the College Board Student Question Bank.

1. What to Expect

- Problem-Solving and Data Analysis: ratios, rates, percentages, tables, graphs, statistics, probability, and reading studies.
- Geometry and Trigonometry: angles, triangles, circles, area and volume, right-triangle trigonometry, and radians.
- Many of these questions are word problems with tables, graphs, or diagrams. Slow down on reading.
- Diagrams are not always drawn to scale. Trust the given numbers, not your eyes.

2. Ratios, Rates, and Units

- A ratio compares two quantities. A rate compares quantities with different units (miles per hour).
- Proportions: cross-multiply. $3/8 = x/20$ gives $8x = 60$, so $x = 7.5$.
- Convert units by multiplying by fractions equal to 1 and cancelling units.

Example: A runner covers 5 miles in 40 minutes. Speed = $5 \div (40/60) = 7.5$ miles per hour.

Example: Convert 90 km/h to meters per second. $90 \times 1000 \div 3600 = 25$ m/s.

Example: A car uses 12 liters to travel 150 km. Efficiency = $150 \div 12 = 12.5$ km per liter.

3. Percentages

- Part = percent × whole.
- Percent change = (new − old) / old × 100.
- Increase by r%: multiply by $(1 + r/100)$. Decrease by r%: multiply by $(1 - r/100)$.
- Successive changes multiply. They do not add.

Example: A price rises from $80 to $92. Change = 12 / 80 = 15% increase.

Example: A 20% increase followed by a 20% decrease. $1.2 \times 0.8 = 0.96$, which is a 4% decrease overall.

Example: After a 25% increase, a value is 90. The original is $90 \div 1.25 = 72$.

4. Tables and Conditional Probability

- In a two-way table, the phrase given that tells you which group becomes the denominator.

Example table: Grade 11 has 40 students (18 take physics, 22 do not). Grade 12 has 40 students (30 take physics, 10 do not). That is 80 students and 48 physics students in total.

- Probability a random student takes physics: 48 / 80 = 3/5.
- Probability that a physics student is in grade 12: 30 / 48 = 5/8.

5. Scatterplots and Lines of Best Fit

- The slope of the line of best fit is the predicted change in y for each 1-unit increase in x.
- The y-intercept is the predicted y when x = 0. It may not make sense in context.
- Predictions inside the data range are interpolation. Predictions far outside are extrapolation and are less reliable.
- A residual = actual value − predicted value.
- Association is not causation.

Example: A line of best fit for quiz score is y = 4.2x + 55, where x is hours studied. Each extra hour of study is associated with a predicted 4.2 points higher. For x = 5, the predicted score is 4.2(5) + 55 = 76.

6. Statistics

- Mean = sum ÷ count. Median = middle value when ordered. Mode = most frequent. Range = largest − smallest.
- Outliers pull the mean toward them but barely move the median.
- A larger spread means a larger standard deviation. Compare how far the values sit from the center, not how large they are.
- A mean from a frequency table: multiply each value by its frequency, add, then divide by the total frequency.

Example: Data 4, 6, 6, 9, 15. Mean = 40 ÷ 5 = 8. Median = 6. Mode = 6. Range = 11. Adding a value of 100 raises the mean a lot but the median only slightly.

Example: Scores of 1, 2, and 3 occur 4, 3, and 3 times. Mean = (4 + 6 + 9) ÷ 10 = 1.9.

Example: {10, 10, 10, 10} has a standard deviation of 0. {2, 6, 14, 18} has the same mean but a much larger standard deviation.

7. Probability

- Probability = favorable outcomes ÷ total outcomes.
- Independent events: multiply. P(A and B) = P(A) × P(B).
- At least one = 1 − P(none).

Example: The chance of rain is 0.3 on each of two independent days. P(rain both days) = 0.09. P(at least one rainy day) = 1 − 0.7 × 0.7 = 0.51.

8. Sampling and Studies

- A random sample lets you generalize to the population it was drawn from.
- Random assignment of subjects to groups allows a cause-and-effect conclusion. Without it, you can claim only association.
- Margin of error: if a poll finds 52% with a margin of error of 3%, the true value is plausibly between 49% and 55%.
- A larger sample usually gives a smaller margin of error.
- Watch for bias such as voluntary response and convenience samples.

9. Lines, Angles, and Triangles

- Vertical angles are equal. Angles on a straight line add to 180°.
- With parallel lines and a transversal: corresponding angles and alternate interior angles are equal. Same-side interior angles add to 180°.
- Triangle angles add to 180°. An exterior angle equals the sum of the two remote interior angles.
- Polygon interior angles add to (n − 2) × 180°.

Example: A triangle has angles 50°, 60°, and x. x = 180 − 110 = 70°.

Similar and right triangles

- Similar triangles have equal angles and proportional sides. If the side ratio is a:b, the area ratio is a2:b2.
- Pythagorean theorem: a2 + b2 = c2. Common triples are 3-4-5, 5-12-13, 8-15-17, and 7-24-25.
- Special triangles: 45-45-90 has sides x, x, x√2. 30-60-90 has sides x, x√3, 2x.
- Example: A right triangle has legs 9 and 12. It is a 3-4-5 triangle scaled by 3, so the hypotenuse is 15.

Example: Triangle ABC is similar to DEF. AB = 6, DE = 9, and BC = 8. The scale factor is 1.5, so EF = 12.

10. Circles

- Area = πr2. Circumference = 2πr.
- Arc length = (θ/360) × 2πr. Sector area = (θ/360) × πr2.
- An inscribed angle is half of the arc it cuts off. A tangent line is perpendicular to the radius at the point of tangency.
- Equation: (x − h)2 + (y − k)2 = r2, with center (h, k).

Example: A circle has radius 6 and a 60° sector. Sector area = (60/360) × π × 36 = 6π. Arc length = (1/6) × 12π = 2π.

Example: Find the center and radius of x2 + y2 − 6x + 4y − 3 = 0. Complete the square: (x − 3)2 + (y + 2)2 = 3 + 9 + 4 = 16. The center is (3, −2) and the radius is 4.

11. Area and Volume

- Rectangle = lw. Triangle = 1⁄2 bh. Trapezoid = 1⁄2 (b1 + b2) h.
- Rectangular prism = lwh. Cylinder = πr2h. Sphere = (4/3)πr3. Cone = (1/3)πr2h.
- Convert units before you compute, and remember that volume units are cubed: 1 m3 = 1,000,000 cm3.

Example: A cylinder has radius 3 and height 10. Volume = π × 9 × 10 = 90π.

12. Right-Triangle Trigonometry and Radians

- SOH CAH TOA: sin = opposite / hypotenuse, cos = adjacent / hypotenuse, tan = opposite / adjacent. sin(x) = cos(90° − x).
- Radians: 180° = π. Useful values: 30° = π/6, 45° = π/4, 60° = π/3, 90° = π/2.
- To convert degrees to radians, multiply by π/180.
- On the unit circle, the point at angle θ is (cos θ, sin θ).

Example: A right triangle has opposite side 5 and hypotenuse 13. The adjacent side is 12. sin = 5/13, cos = 12/13, tan = 5/12.

Example: Convert 120° to radians. 120 × π/180 = 2π/3.

13. Strategy

- Redraw and label diagrams. Write the given numbers on them.
- Underline exactly what the question asks for.
- For tables and graphs, read the title, labels, and units first.
- Use Desmos for regression, for plotting functions, and for checking your algebra.
- When a diagram has no scale given, do not measure by eye.

14. Common Traps

- Adding percent changes instead of multiplying them.
- Using the wrong denominator in a conditional probability.
- Forgetting to square the scale factor for area, or cube it for volume.
- Confusing radius and diameter.
- Giving the answer in the wrong unit.

15. Practice Drills

1. Convert 54 km/h to meters per second.

2. A price of $120 rises 25%, then falls 20%. What is the final price?

3. What is the median of 3, 5, 5, 7, 10?

4. A fair die is rolled twice. What is the probability both rolls are 6?

5. What is the probability of at least one head in 3 fair coin flips?

6. Is a triangle with sides 7, 24, and 25 a right triangle?

7. What are the center and radius of $(x + 1)2 + (y − 4)2 = 25$?

8. In a right triangle, $\sin \theta = 3/5$. What is $\cos \theta$?

9. Convert 150° to radians.

10. A poll of 400 people finds 48% in favor, with a margin of error of 5%. Is it plausible that a majority favors the measure?

Answers

1. 15 m/s. $54 \times 1000 \div 3600 = 15$.

2. $120. $120 \times 1.25 \times 0.8 = 120$.

3. e. It is the middle value.

4. 1/36.

5. 7/8. $1 − (1/2)3 = 7/8$.

6. Yes. $49 + 576 = 625 = 252$.

7. Center (−1, 4) and radius 5.

8. 4/5. The hypotenuse is 5, the opposite side is 3, so the adjacent side is 4.

9. $5\pi/6$. $150 \times \pi/180 = 5\pi/6$.

10. Yes. The plausible range is 43% to 53%, which includes values above 50%.

Next step: work through 30 official Problem-Solving and Data Analysis and Geometry and Trigonometry questions in the College Board Student Question Bank, and keep a list of every unit, denominator, or diagram mistake you make.